$$\pi 3.^{24} R = \frac{\pi[3^{12}-1]}{2\log_e 3}. \quad R = \frac{3^{12}-1}{2.3.^{24}\log_e 3}.$$

Also solved by *A. H. HOLMES*, *J. SCHEFFER*, and *B. F. YANNEY*.

ERRATA. In last issue, page 120, line 4 from bottom, for "$\rho = \frac{\theta^2}{c^2}$" read, $\rho^2 = \frac{\theta^2}{c^2}$.

PROBLEMS.

55. Proposed by GEORGE LILLEY, Ph. D., LL. D., Principal of Park School, 394 Hall Street, Portland, Oregon.

A horse is tethered by a rope, a feet long, fastened to a post in a circular fence enclosing a circular piece of ground b feet in diameter. If the horse is outside of the fence over how much ground can he feed? If he is inside the fence over how much ground can he feed? $b > a$ in each case.

56. Proposed by Prof. B. F. BURLESON, Oneida Castle, New York.

Find (1) the length s of the closed curve of the cardioid; (2) its area A; (3) if made to revolve about its axis $2a$, find the maximum longitudinal circumference C of the solid generated; (4) find the surface K of the same; (5) its volume V; (6) the distance x_0 of the center of gravity of the solid from the origin O; and (7) the distance g_0 of the center of gravity of the plane curve from the origin O.

MECHANICS.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

31. Proposed by O. W. ANTHONY, M. Sc., Professor of Mathematics in New Windsor College, New Windsor, Maryland.

A perfectly elastic, but perfectly rough mass M, and radius R, rotating in a vertical plane with an angular velocity ω, is let fall from a height, a, upon a perfectly elastic but perfectly rough horizontal plane. Determine the motion of the body after striking the plane. What will be its ultimate motion?

II. Solution by G. B. M ZERR, A. M., Ph. D., Professor of Mathematics and Applied Science, Texarkana College, Texarkana, Arkansas-Texas.

Let V be the vertical velocity of the center just before impact; u, v, the horizontal and vertical velocities of the center just after the first impact; ω_1 the

angular velocity after first impact ; u' the velocity of the center just before the second impact ; u_1, ω_2 the values of u, ω_1 just after the second impact, k the radius of gyration.

The equations of motion for first impact are

$$(v+V)(k^2+R^2)=2V(k^2+R^2) \ldots\ldots (1).$$

$$u(k^2+R^2)=\omega Rk^2 \ldots\ldots (2).$$

The geometrical condition for no sliding is

$$u-R\omega_1=0 \ldots\ldots (3),$$

but $V=\sqrt{2ag}$, $k^2=\frac{2}{5}R^2$.

$$\therefore\ v=\sqrt{2ag},\ u=\tfrac{2}{7}R\omega,\ \omega_1=\tfrac{2}{7}\omega,\ u'=\sqrt{v^2+u^2}=\tfrac{1}{7}\sqrt{4\omega^2R^2+98ag}.$$

If β be the angle the center of the sphere makes with the plane just after impact we easily get

$$\cos\beta=\frac{u}{\sqrt{u^2+v^2}}=\frac{u}{u'},=\frac{2R\omega}{\sqrt{4\omega^2R^2+98ag}}.$$

Thus the motion is determined after striking the plane. Let F be the impulse arising from friction, then the equations of motion for second impact are,

$$Mu_1=Mu'\cos\beta+F \ldots\ldots (4),$$

$$\tfrac{2}{5}MR^2\omega_2=\tfrac{2}{5}MR^2\omega_1-RF \ldots\ldots (5),$$

and the geometrical condition $u_1-R\omega_2=0 \ldots\ldots (6)$.

$$\therefore\ F/M=-\tfrac{2}{7}(u'\cos\beta-R\omega_1),\ u_1=R\omega_2=\tfrac{5}{7}u'\cos\beta+\tfrac{2}{7}R\omega_1,$$

but $u'\cos\beta=R\omega_1$, $\therefore$ $F/M=0$, and no impulsive friction is called into play after the first impact. Hence the center of the sphere describes the same parabola after each impact and the ultimate motion is the same as that after striking the plane.

III. Solution by the PROPOSER.

Each motion of the sphere may be considered, in its reactionary effect, separately. The motion of translation will cause the sphere to rebound after each impact to its original altitude. The time taken to attain the altitude a will be $t=\sqrt{\dfrac{2a}{g}}$.

The effect of the motion of rotation may be considered in this way : Let a rotating sphere be brought into contact with a plane slowly. The sphere will, of course, roll along the plane. The energy of translation and rotation being equal to the original energy, E, we shall have the same result in the case

under consideration, that is, we shall have the same velocity parallel to the plane, and the same angular velocity as if the sphere were in contact *with* the plane, because there is no slipping at the instance of contact.

Let v_1=velocity parallel to the plane. Then $\frac{v_1}{R}$=new angular velocity $=\omega_1$.

$$E=\tfrac{1}{5}MR^2\omega^2.$$

$$\text{Energy of translation}=\tfrac{1}{2}Mv_1^2.$$

$$\text{New energy of rotation}=\tfrac{1}{5}MR^2\omega_1^2=\tfrac{1}{5}Mv_1^2.$$

$$\therefore\ \tfrac{1}{5}MR^2\omega^2=\tfrac{1}{2}Mv_1^2+\tfrac{1}{5}Mv_1^2. \quad \text{Whence,}$$

$$v_1^2=\tfrac{2}{7}R^2\omega^2.$$

$$\therefore\ v_1=\sqrt{\tfrac{2}{7}}R\omega, \text{ and}$$

$$\omega_1=\sqrt{\tfrac{2}{7}}\omega.$$

The distance which the sphere will move parallel to the plane while it is attaining its highest altitude will be $=tv_1=2\sqrt{\frac{a}{7g}}R\omega$.

From these data, knowing that the curve will be a parabola, we obtain

$$y^2=\frac{4R^2\omega^2}{7g}x,$$

the highest point in the origin. The distance between first and second impact is $4\sqrt{a/7g}\,R\omega$. As to the subsequent motion, we have the equation of energy

$$\tfrac{1}{2}Mv_1^2+\tfrac{1}{5}Mv_1^2=\tfrac{1}{2}Mv_2^2+\tfrac{1}{5}Mv_2^2, \text{ or } v_2=v$$

and the subsequent parabola will be the same as the first.

PROBLEMS.

37. Proposed by O. W. ANTHONY, M. Sc., Professor of Mathematics and Astronomy, New Windsor College, New Windsor, Maryland.

A thin board of which the elements are given is balanced at the center but inclined at an angle. A sphere of known dimensions is put directly above the point of suspension and liberated. Find the motion of the system. That is, find (a) the time until the sphere leaves the board, (b) the ultimate angular velocity of the board.

38. Proposed by B. F. FINKEL, A. M., Professor of Mathematics and Physics, Drury College, Springfield, Missouri.

A prolate spheriod of revolution is fixed at its focus; a blow is given it at the extremity of the axis minor in a line tangent to the direction perpendicular to the axis major. Find the axis about which the body begins to rotate. [From *Loudon's Rigid Dynamics.*]